Wheel of Theodorus

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CONCEPT: Geometry
SKILLS: Applying the Pythagorean theorem in an art context, noting pattern and structure
MATH CONTENT STANDARDS: 8.G.7
MATH PRACTICE STANDARDS: 4, 6, 7, 8
GRADES: 6–10
MATERIALS: For each student: printed instructions, one 3 x 5 index card, white practice paper, large white art paper, pencil, eraser, straightedge, colored pencils or watercolors, and rulers for optional **Activity Sheet** (p. 37) to be used for upper grades.

BACKGROUND

One effective exploration of irrational lengths also creates a beautiful—even inspiring—spiral design named the Wheel of Theodorus or the Spiral of Theodorus. By displaying the spiral arrangements of a series of right triangles, students obtain precise measurements of irrational line segments. According to Boaler (2017), "When students think of math visually as well as with numbers and symbols, they are crossing the brain, using different pathways, and that has been found to increase the power of math learning."

Theodorus, a member of the Society of Pythagoras, taught mathematics and was a tutor of Plato. Plato cited Theodorus' work with the roots of non-square integers from 3 to 17 in his dialogue, Theaetetus. Although the original work of Theodorus himself was lost, it was preserved for us by Plato.

This activity presents a natural and mutually beneficial opportunity for collaboration between art and mathematics. Students can complete this activity within two class sessions. Allow sufficient time for students to practice the initial procedures and explore the Pythagorean theorem in the first session. In the second session, allow time for students to apply their art techniques in the chosen medium.

Art by Lexi Katsones

PREPARATION

Provide students with directions, index cards, white practice paper, large white paper, pencils, erasers, and straightedges. For upper grades, provide activity sheets and rulers.

PROCEDURE

Constructing the Wheel

Constructing the Wheel of Theodorus is an exercise that requires the student to use the valuable skills of precision and attention to detail that are found in the Mathematical Practices. In this approach, we use a method that simplifies the process by using the corner of an index card to "construct" the right angles required. A simple 3" x 5" index card works very well for this purpose. Teachers provide the following instructions to students and monitor the progress of their work carefully, especially as they begin to create the appropriate triangles.

1. Using one corner of the index card, mark equal lengths on adjacent sides of the corner (***Figure 1***). For upper-grade students who will be calculating the exact length of each hypotenuse using the Pythagorean Theorem, it is helpful to choose a specific length, such as three centimeters, to mark on the sides of the index card. This precise measurement will make the measuring of the hypotenuse and the calculations easier as the lengths of each successive hypotenuse grow longer.

Figure 1

2. Place the corner of the index card in the approximate center of your white practice paper. Trace the right angle, and carefully mark the lengths of the sides of the angle (***Figure 2***).

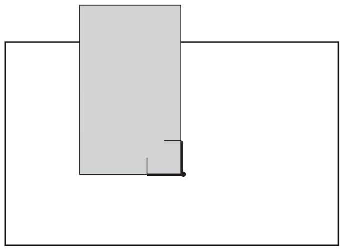

Figure 2

3. Use a straightedge to draw the hypotenuse of the isosceles triangle (***Figure 3***).

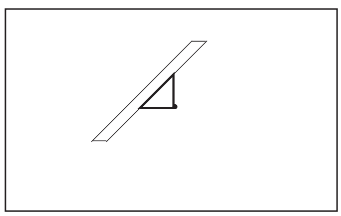

Figure 3

4. Next, use the right angle on the index card and align it with the vertex that connects the hypotenuse to one leg of the first right triangle. As precisely as you can, redraw the original unit length at a right angle to this hypotenuse (***Figure 4***).

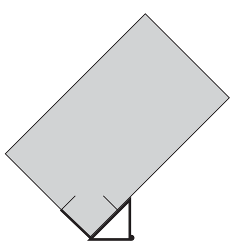

Figure 4

5. Connect the endpoint of this new side to create a second right triangle with the new hypotenuse that is longer than the first (***Figure 5***).

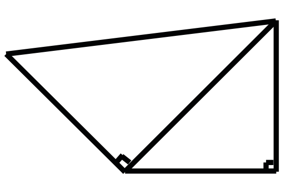

Figure 5

6. The students should make at least three practice triangles on the practice paper before beginning the process on the large white paper.

7. Repeat steps 4 and 5 to create additional triangles that will spiral around the initial triangle, drawing as many triangles as you can manage on your paper. As the spiral grows, you should have 16 triangles before the lines begin to overlap, and all of the triangles have a common vertex at the "center" of the spiral. Each successive hypotenuse will be longer than the previous triangle (***Figure 6***).

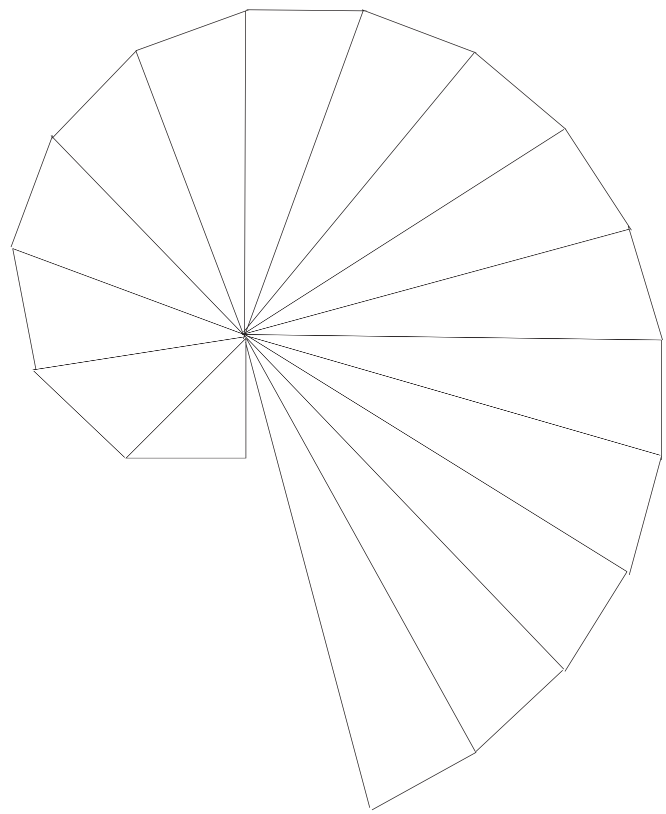

Figure 6

Students may choose to continue to draw additional triangles beyond the original 16, but the triangles will overlap. Artists can choose to have the lines of overlapping triangles not be drawn on top of the earlier triangles. Remind students to clearly align each new hypotenuse into the common vertex of the spiral.

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Checking for Accuracy

After the students have completed at least eight triangles in the spiral, ask the students to stop and check their accuracy. Have students take the distance they had marked off on the index card originally and find a hypotenuse that is the exact double of that distance. The length of the hypotenuse of the third triangle should be exactly double the size of the first side. Then have the students find which triangle's hypotenuse has triple the length of the first side. The eighth triangle's hypotenuse should be triple the length of the first side. Ask students to predict when the triangle's hypotenuse will be four times the original length.

Vzia Romo said, "It was cool how math can be visually appealing."

Art by Akabi Peary

The Pythagorean Theorem

This activity provides a perfect time to teach the Pythagorean theorem, even if students have not studied it formally. Have students calculate the length of each hypotenuse as they draw it using the equation $a^2 + b^2 = c^2$. Have students record their answers in a table (see **Activity Sheet** page 37). For upper grades, have the students simplify the radicals to discover a pattern to find the next consecutive hypotenuse length. The length of the first hypotenuse will be the length of the first segment times the square root of 2. The next hypotenuse will be the original length times the square root of 3, and the following hypotenuse will be times the square root of 4. The pattern will continue for each successive hypotenuse length. As the students work, discuss what the square root of 4 is (i.e., what number times itself equals 4?). Also, encourage student discussions regarding rational lengths of the hypotenuse (for example, the third, eighth, and 15th triangles) and the rest of the lengths of the hypotenuse, which have an irrational length.

Candee Seed's art took a different turn. She said, "It was fun to see how math can be art."

Notes About the Artwork

The artwork accompanying this article is a collaborative effort with the mathematics and art classes at Santa Lucia Middle School students in Cambria, CA. Suzette Morrow, the art teacher, encouraged students to experiment with the color wheel using primary colors to mix secondary and tertiary colors as the colors rotated through the various right triangles forming the Wheel of Theodorus. The upper middle school students used creativity in designing other color theory explorations with the index card method. This activity also ignites the student's interest by connecting the beauty of art with mathematics. Making small changes in approach can both motivate and deepen student's engagement. For more examples of art using the Wheel of Theodorus, search "images of art using the Wheel of Theodorus."

Students experiment with the color wheel using primary colors to mix secondary and tertiary colors.

Art by Aimee Venegas

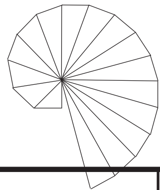
References

Boaler, J. 2017. *Week of Inspirational Math*. Resourced from: **https://www.youcubed.org/resource/classic-wim-week-1-grades-3-4/**

Venters, D., and E. Krajenke Ellison. 1999. *Mathematical Quilts: No Sewing Required*.

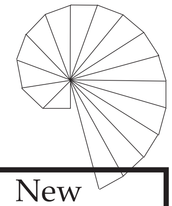
All student art used by parent/guardian permission.

Activity Sheet page 37 . . .



Wheel of Theodorus

Activity Sheet



Right Triangle	Initial Length	New Leg Length	Pythagorean Theorem $a^2 + b^2 = c^2$	New Hypotenuse Length
1	$\underline{3}$ cm	3 cm	$\underline{3}^2 + \underline{3}^2 = c^2$	c = ______
2	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______
3	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______
4	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______
5	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______
6	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______
7	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______
8	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______
9	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______
10	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______
11	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______
12	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______
13	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______
14	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______
15	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______
16	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______
17	______ cm	3 cm	$\_\_\_\_^2 + \_\_\_\_^2 = c^2$	c = ______

Accuracy Test:

1. Calculate the 17th triangle's hypotenuse length, then use a calculator to estimate the approximate length of the hypotenuse to the nearest 10th: __________ cm
2. Use a ruler to measure the length of that hypotenuse to the nearest 10th: __________ cm
3. What is the difference between the two answers to the nearest 10th? __________ cm
4. Was your drawing accurate? Explain: